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Talk- Mathematical thinking
Order[s] in Architecture – 2018W
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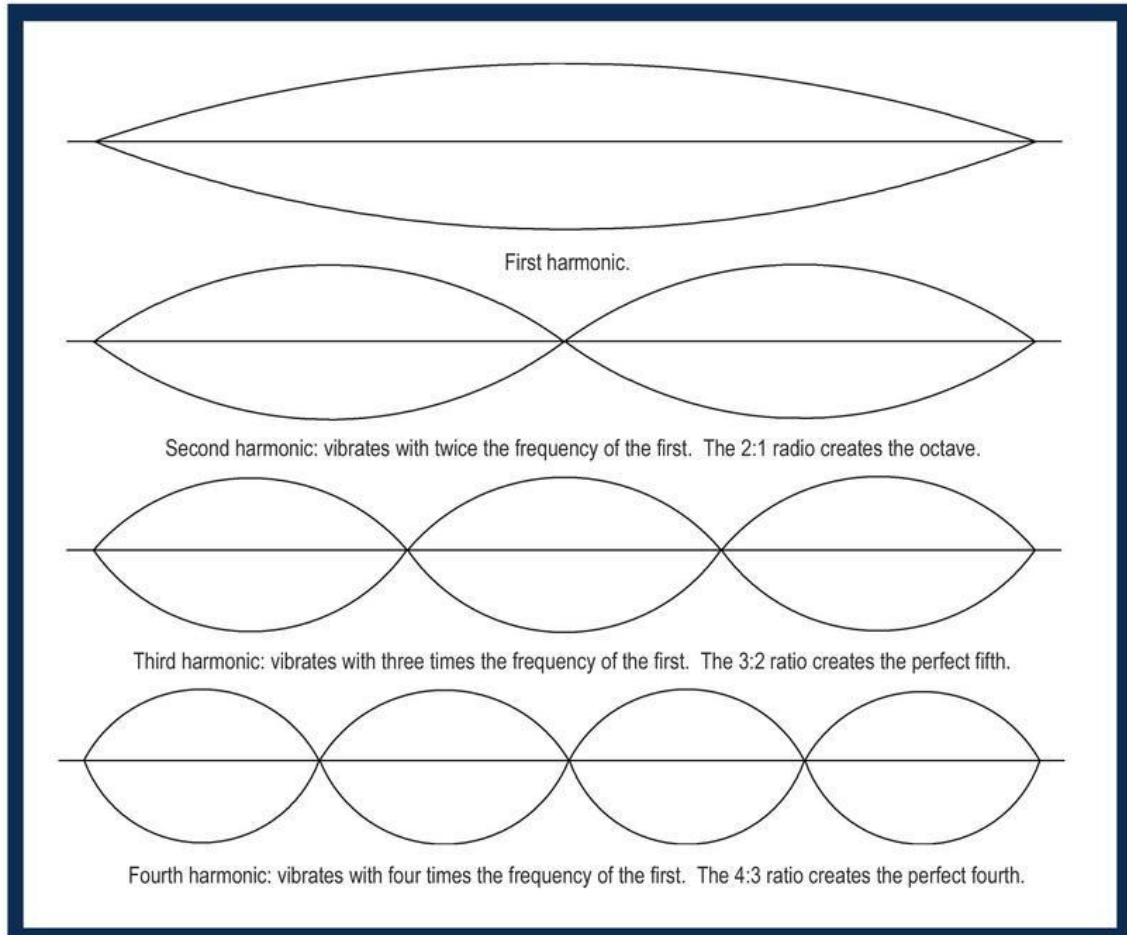
Talk

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The question: Consider the following picture of a sonic chord, or vibrating string, i.e. the standing wave formed by the bow and a string of a violin:

Harmonics of a Vibrating String

A string, fixed at two ends, vibrates in its "normal modes", referred to in music as harmonics.



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- Describe the relation between the length of the string and the frequency. What is the algebraic group structure of the harmonics?
 - How would you characterize the hybrid relation between the bow and the violin string? What is the role of the points of stasis in the above diagram?
 - What bridges and scaffolding would you use to extend the group structure of harmonics to the group structure of frequency ratios?
 - Is it possible to set up a rational scale, or equivalently partition an octave (2:1 ratio) using the fifth (3:2 ratio) as a rational unit?

First, it is about a freezing pattern of harmonic that can be represented as a kind of weaving. Since weaving bring stability, so we can say that it can keep the memory as well. Which I found the most interesting fact in this topic. In order to approach this fact, we should first search how music works through harmonic. The theory of standing wave and Monochord will be very helpful to be listed in the beginning of this process. It is a type of wave which is not travelling in space. Space is fixed here, so we have two bridges and within these two bridges we have a chord. This chord can vibrate. In this kind of wave, space doesn't play any role. Everything is about the domain of time. What we first notice is, if we reduce the chord into the half, the frequency will be doubled, and this is what we call the second harmonic. In other words, we have here a reverse relationship between the frequency and the length of the string. We realize also that all possible harmonics appears as insider multipoles of the fundamental harmonic. This gives us also the possibility to express the relationship between musical intervals and ratios, that I will discuss in the next few lines.

Now let us discuss the issue from the algebraic group structures point of view. At the first glance, it seems as if we are thinking in terms of linear relations, but in fact it is circular one. We basically have three circles here. The interval of the fourth is inversed to the interval of the fifth with respect to the whole octave. Furthermore, Intervals can be joined to something else or can be subtracted. That's mean the relation between musical intervals and ratios is a logarithms nature, because the logarithm function transforms multiplication to addition and division to subtraction. This is how we make sense of musical intervals in terms of ratio through the logarithmic function. This is what we also call the structural relation between geometrical intervals and ratios.

In order to discuss the hybrid relation between the bow and the violin string, we should first know that the desired sounding point will generally move closer to the bridge as the string is stopped to a shorter sounding length. Furthermore, the amount of hair in contact with the string could be vary by tilting the bow stick more or less away from the bridge. Basically, the bow needs to remain perpendicular to the strings for the entire bow stroke. The key element in this regard is the length of the bow arm.

To understand the function of the points of stasis, we let the string vibrate and then hold the middle of it (one-part stop vibrating), the other move, the frequency will get doubled. In specific points which don't move but are not static, the vibration will be reflected. These points are invariance. The process here is a kind of homeostatic process.

It is important to be mentioned that in mathematics we don't really solve problems rather we embrace problems. In other words, when we have a problem in the upper level. We can't go directly from the initial point to the final rather we need to make a scaffolding or to build a bridge in the both sides. By this one round trip, we have managed to embrace the obstacle. This process involves two levels that communicate to each other. We consider this process (encoding/ decoding bridges) as a way of finding a solution for any problem. Moreover, in harmony we don't have subject and object that's mean we lose the ideology, and that's what in fact means harmony. Hence, in order to find the suitable bridges and scaffolding that could help us extending the group structure of harmonics to the group structure of frequency ratios, we need first to know more about the notion "Logos". Logos refers to the universal objective schema of Harmonic Communication of nature with us. It is been always established and assigned to it the Equivalence class of 1=one True in binary Partition. And it refers also to the acoustic domain. Thus, the relation here is Logos, harmonic relation and Arithmos (spectrum). Transforming between harmonic and Arithmos (spectrum) is going through Algebra. Ascending via the Geometric Progression and Descending back via the Arithmetic Progression. So, from morphology of choreography to scenography. From scenography to ichnography via orthography.

It is important to state that it is wrong to think about rational through geometry, but through harmony. Irrational numbers don't come from harmony, but they come from geometry.

It is also important to differentiate between these two different notions: Harmonic and Arithmetic. In order to make it clear we can check these two equations:

If $(2\alpha + \beta) / (\alpha + \beta) = 8$ that's mean Harmonic.

But if $(\alpha + \beta) / 2 = 9$ that's mean Arithmetic.

This process is basically a search of a multiplicative common measure, so we can check these ratios first and clarify what do "tone", "diesis" and "limma" mean:

- Three small ratios corresponding to consonant musical intervals: $2/1$, $3/2$ and $4/3$.
- $2/1 = [3/2] [4/3]$
- $3/2 = [4/3] [9/8]$
- $9/8$ corresponds to a new musical interval called a "tone" which is the smallest musical interval, but it is not "tone" the required common measure, because we can keep dividing and we need smaller and smaller musical intervals. Because the process of dividing is able to be continued, so we don't have a common scale.
- $4/3 = [9/8] [32/27]$
- $[32/27] > [9/8]$, hence: $32/27 = [9/8] [256/243]$.
- $256/243$ corresponds to a new musical interval called a "diesis" or "limma".
- $4/3 = [9/8] 2 [256/243]$.

Although they don't have common measure, we can build scales by using two different measures. We basically have Diesis and Tone and the Diesis is almost the half of the tone (the square root of the tone). For some musician they use the Diesis as a unit. Through these two different units we build scale. If we complete in this process, we come to the [Comma] = $312/219$ which is translated to $[12 \text{ intervals of the fifth}] = [7 \text{ intervals of the octave}] \bmod [\text{Comma}]$.

Furthermore, by using this small ratio (octave) we can build different chords. It was a revolution to change the limma that has seventh chords. They took two chords and instead of joining them together the intervenes between them and they built an octave world. Octave = $[2/1] = [\text{Tone} 5/1] [\text{Diesis} 2/1]$. They make up a scale through two different measures. So, the musical intervals of the (Octave, fifth, fourth) have their corresponding logos in the ratios (2:1, 3:2, 4:3) frequencies. What we can here also conclude is that the incommensurability of the diagonal with the side in geometry comes from music. Thus, what we describe as Harmonics in music is been noticed when we divide the string into sections in the ratios 2:1 and 3:2 yields the intervals of a perfect octave and perfect fifth, respectively.